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Characterization of Intermodulation Distortion in Multicarrier Transmission Systems

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Abstract — **This paper proposes the analysis of intermodulation distortion in a multicarrier system, using a method which exploits the relationship between measured non-linear performance and the signal characteristics of the second- and third-order distortion products. This method, based on a power series expansion, is applicable to situations involving many non-harmonically related tones, where harmonic balance analysis becomes impractical.**

I. INTRODUCTION

Modern communication systems employ active devices, which are required to operate over a wide bandwidth, in the presence of several carriers with wide amplitude range and complex signal modulation schemes. Examples include base station amplifiers in cellular systems and video amplifiers used in hybrid fiber coaxial (HFC) transmission systems for CATV delivery. In HFC systems, the amplifiers need to exhibit exceptional linearity in order to limit the video image impairment caused by undesirable composite second order (CSO) and composite triple beat (CTB) distortion [1]. This requirement is compounded by the desire to transmit several channels (on several RF carriers) of video and digital programming with high output levels on the amplifiers. Depending on the frequency plan used in a particular application, the number of channels contributing to potential degradation of linearity may be as high as 150 to 200. Therefore, it becomes very important to characterize, preferably with a mathematical model, the intermodulation distortion in a wide-bandwidth multicarrier transmission environment.

The conventional approaches to non-linear amplifier characterization, such as the Volterra series approach or the harmonic balance method [2], have been found to be quite effective for a few tones, but the computational burden becomes prohibitively large when tens or hundreds of signals are involved. In the United States' NTSC cable television system, because of FCC regulations on frequency usage, the carriers are not even harmonically related [1]. Therefore, other methods need to be employed for the study of intermodulation distortion in such situations. Alley and Kuo [3] model the second-order distortion in HFC systems using the concepts of steepest descent search and the "ambiguity function" to determine the optimum carrier phase for each tone. In practice, although the carrier phase at the laser input may be controlled to the desired optimum, the non-linearities involved in the laser and other active devices further down-link, cause the trunk amplifiers to experience at best random phase fluctuations at their input. Germanov [4] derived a formulation to relate the second-order and the third-order intermodulation constants, measured under small-signal conditions, to the system CSO and CTB. Although this approach simplifies the analysis and defines the composite distortion using the conventional non-linear formulation applicable to a few input tones [2], its accuracy is limited in practice because the intercept points drift with frequency.

In this paper, we propose a method to analyze intermodulation distortion in a multicarrier system, which exploits the relationship between measured non-linear performance and the signal characteristics of the second- and third-order distortion products. Because we use measured data to develop the non-linear model, the proposed method does not suffer from the practical limitations in [3], [4]. In general, the power in the composite distortion products at a given input signal level is determined by *both* the non-linearities of the transmission system and the composite signal waveform being transmitted. However, we assume that the measurement accurately captures the system non-linearity, and focus on developing a model, which describes the impact of such non-linearity on the composite signal degradation. Without loss of generality, we consider a multi-channel HFC link amplifier for illustration, but the method can be applied to any device or system where multiple intermodulation products need to be characterized.

In Sec. II, we present the basic mathematical formulation of the proposed approach and briefly describe the frequency plan used in the paper. Sec. III discusses the signal characteristics of the distortion products, and shows the procedure to determine the non-linearity coefficients from measured data. Sec. IV describes data measured on

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the amplifier at a few frequencies and applies the proposed method to describe its non-linear performance.

II. Mathematical Formulation

An N-channel multi-carrier signal can be described as

$$x(t)=\sum_{n=1}^{N} a_n \cos(\omega_n t+\phi_n) \qquad (1)$$

where a_n is the amplitude, $\omega_n = 2\pi f_n$ is the radian frequency, and ϕ_n is the phase, of the n[th] (unmodulated) carrier. This signal passes through an amplifier and generates an output waveform containing second- and third-order distortion products:

$$y(t)=G_1 x(t)+G_2 x^2(t)+G_3 x^3(t) \qquad (2)$$

where G_1, G_2 and G_3 are the gain, second- and third-order non-linearity coefficients of the amplifier. All three coefficients are in general time-variant. The distortion products are given by [2]

$$\begin{aligned} x^2(t) &= \sum_{n=1}^{N}\frac{a_n^2}{2}+\sum_{n=1}^{N}\left(\frac{a_n^2}{2}\right)\cos(2\omega_n t+2\phi_n) \\ &+\sum_{\substack{n=1\\ n\neq m}}^{N}\sum_{m=1}^{N} a_m a_n \cos\left[(\omega_m \pm \omega_n)t+(\phi_m \pm \phi_n)\right] \end{aligned} \qquad (3)$$

$$\begin{aligned} x^3(t) &= \frac{3}{4}\sum_{n=1}^{N} a_n^3\cos(\omega_n t+\phi_n)+\frac{1}{4}\sum_{n=1}^{N} a_n^3\cos(3\omega_n t+3\phi_n) \\ &+\frac{3}{2}\sum_{\substack{n=1\\ n\neq m}}^{N}\sum_{m=1}^{N} a_m^2 a_n \cos(\omega_n t+\phi_n) \\ &+\frac{3}{4}\sum_{\substack{n=1\\ n\neq m}}^{N}\sum_{m=1}^{N} a_m^2 a_n \cos\left[(2\omega_m \pm \omega_n)t+(2\phi_m \pm \phi_n)\right] \\ &+\frac{3}{2}\sum_{\substack{p=1\\ p\neq n\neq m}}^{N}\sum_{n=1}^{N}\sum_{m=1}^{N} a_m a_n a_p \cos\left[\begin{array}{l}(\omega_m \pm \omega_n \pm \omega_p)t \\ +(\phi_m \pm \phi_n \pm \phi_p)\end{array}\right] \end{aligned} \qquad (4)$$

Henceforth, for illustration, we assume that the signal in (1) represents the NTSC incrementally related carrier (IRC) channel plan, consisting of 79 analog channels. This plan, shown in (5), excludes channels in the FM band to minimize interference in this band. Notice that the frequencies are not harmonically related.

$$f_n\ (\text{MHz})=\begin{cases} 55.25+(n-1)6, & n=1,2,3 \\ 77.25+(n-4)6, & n=4,5 \\ 109.25+(n-6)6, & n=6,\cdots,79 \end{cases} \qquad (5)$$

The waveform $x(t)$ for this channel plan is shown in Fig. 1, assuming coherent phasing of all channels ($\phi_n = 0$). The waveform displays impulsive energy bursts at periodic intervals separated by the period, $T=1/6e6=0.167\ \mu s$. The power spectrum of this waveform exhibits the familiar $\sin(Nx)/\sin x$ envelope, with a peak-to-average ratio of 19 dB [5].

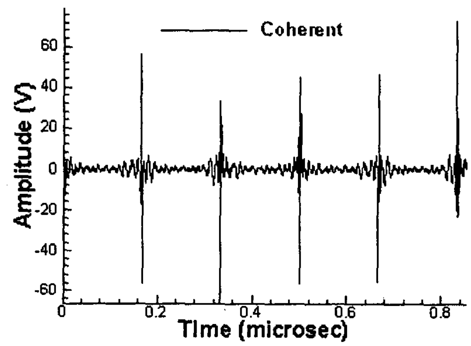

Fig. 1. Time waveform for coherently phased IRC plan.

In reality, the carriers are not co-phasal, but randomly phased. Assuming phase ϕ_n to be uniformly distributed over $[-\pi,\pi]$, the corresponding waveform (Fig. 2), does not exhibit the periodic peaks present in the coherent case. The spectrum of the randomly phased signal also lacks any periodic pattern (Fig. 3). Interestingly, due to the large channel number, the power spectral peak-to-average ratio approaches the co-phasal limit of 19 dB!

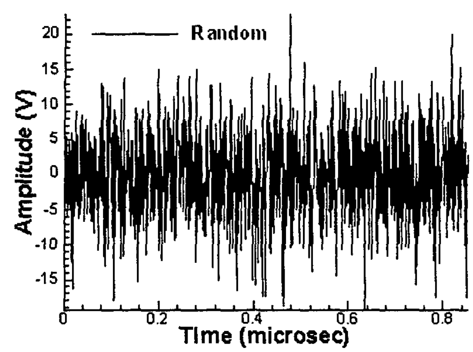

Fig. 2. Time waveform for randomly phased IRC plan.

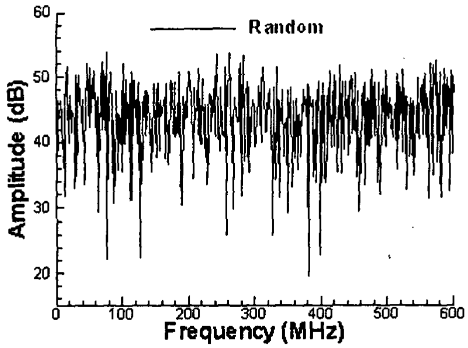

Fig. 3. Spectrum for the randomly phased waveform.

III. Distortion Characterization

A. Measurement

The distortion performance in a multi-carrier system is specified as composite second order and composite triple beat products, and is measured with unmodulated carriers. Since modulation usually improves the CSO and CTB figures [1], the measurement refers to the worst-case values. The CW carriers mix (beat) with each other through the amplifier non-linearity, producing individual second- and third-order products exemplified in (3) and (4), respectively. CSO is the composite (sum) of all the second-order discrete individual beats at the output, and CTB is the composite of all the third-order beats. In the IRC frequency plan of (5), the CSO beats, measured as a group, fall at $(f \pm 1.25)$ MHz. The CTB beats fall around the intermodulation frequency, f.

The measurements are made at few frequencies using a multi-tone generator and a spectrum analyzer. The reader is referred to [1] for the procedural details. CSO is measured as the ratio, in dB, of the peak of the RF signal to the average level of the cluster of second-order products centered ± 1.25 MHz around the carrier. A similar procedure applies to CTB measurements, but the average is taken on clusters centered around the carrier. If the carriers are phase-locked, the beats would also be coherent, and the predicted CSO/CTB values will be unusually large. Therefore, random phase offset is introduced in the measurement system on all the carriers excited for the intermodulation study. As hinted in Fig. 3, the spectrum of the beats resembles noise because it is made up of many carriers. Assume that all the carriers are excited with uniform amplitude ($a_n = a_c, n = 1, \cdots, N$).

B. Beat Count

Since all mixing frequencies have the same excitation amplitude, the composite beat power measured at a given frequency is directly related to the *number of contributing discrete beats*. Therefore, accurate counting of the mixing products is important. For *uniform* channel spacing, approximate expressions are available for the number of significant beats contributing to CSO and CTB at any frequency [1]. However, for the non-uniform IRC frequency plan in (5), these equations are not accurate. Using (3) and (4), we have written a computer program to calculate the CSO and CTB beats produced at a given intermodulation frequency. The beat count data will be presented in Sec. IV. For CSO, it follows from (3) that the sum and difference frequency beats, $f_m \pm f_n$, fall within the IRC frequency band, while the second harmonics fall outside, and are 6 dB lower. For CTB count, from (4), we note that the $f_m \pm f_n \pm f_p$ beats are dominant, being 15.6 dB stronger than the third harmonics $3f_n$, and 6 dB stronger than the $2f_m \pm f_n$ beats (but these are few and can be neglected).

C. Determination of Non-Linearity Coefficients

The system distortion is characterized by the second- and third-order non-linearity coefficients, introduced in (2). We now show how these can be obtained from the measured CSO and CTB at the intermodulation frequency, f. Since the phases are independent, uniformly distributed random variables, the peak voltage in the non-linear beat output may be written in terms of sum of squares of the individual beat amplitudes. The ratio of the non-linear voltage peak magnitude to the carrier's peak voltage output equals the measured CSO and CTB, and relates to the desired non-linearity parameters, $D_2(f)$ and $D_3(f)$, respectively [6]:

$$\frac{\sqrt{\sum_{n=1}^{N_{CSO}} \left[a_i a_j G_2(f)\right]^2}}{a_c G_1(f_c)} = D_2(f)\sqrt{N_{CSO}} \triangleq CSO \quad (6)$$

$$\frac{\sqrt{\sum_{n=1}^{N_{CTB}} \left[\frac{3}{2}\, a_i a_j a_k G_3(f)\right]^2}}{a_c G_1(f_c)} = D_3(f)\sqrt{N_{CTB}} \triangleq CTB \quad (7)$$

Since all the amplitudes are equal (to a_c), it follows that

$$D_2(f) = \frac{G_2(f)a_c}{G_1(f_c)}, \quad D_3(f) = \frac{3}{2}\frac{G_3(f)a_c^2}{G_1(f_c)} \quad (8)$$

Thus, using the measured CSO and CTB in the second equalities of (6) and (7), we may calculate D_2 and D_3. Unlike the measured values, the latter two are independent of the channel plan.

IV. RESULTS

We have measured the CSO and CTB distortion on a 20 dB hybrid gain block, intended for linear operation over 40 to 900 MHz with a maximum output of 10 W. The non-linear products were measured at 6 frequencies spanning the IRC channel plan. In this section we present the non-linear parameters derived from our model. Fig. 4 displays the beat product count as a function of intermodulation frequency, computed *exactly* using expressions derived from (3) and (4). Note that the beat count is independent of the measurement. It depends only on the channel plan. It is observed that the third-order products outweigh the second-order count by two orders of magnitude. However, at the band edges, the latter cannot be ignored. The CSO_- beats fall 1.25 MHz below the carrier while CSO_+ beats fall 1.25 MHz above it.

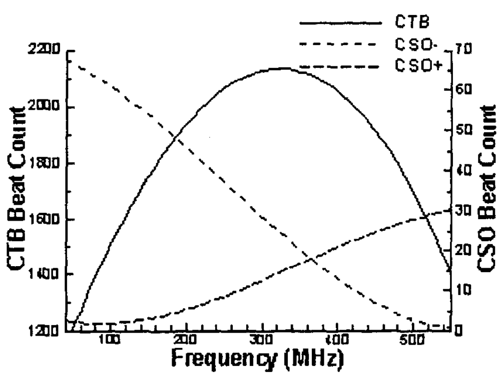

Fig. 4. Calculated beat count for the IRC channel plan.

Fig. 5 plots the non-linearity coefficients, computed from (6) and (7), using the measured CSO, CTB, and the number of beats calculated for the IRC plan in (5). All the excited carriers are maintained at 44 dBmV flat output level over 55 to 550 MHz. Parameter D_3 is fairly constant across the band. Realizing that (6) and (7) are voltage relationships, these coefficients can be interpreted as the normalized (or per beat) equivalents of the measured CTB or CSO. The normalized value should be independent of the channel plan [4]. In order to verify this observation, the amplifier distortion is measured with the 131-channel plan, spanning 55 to 860 MHz. The output level is maintained at 44 dBmV. The non-linearity constants derived from this data are also plotted in Fig. 5. Excellent agreement is observed between the two sets of

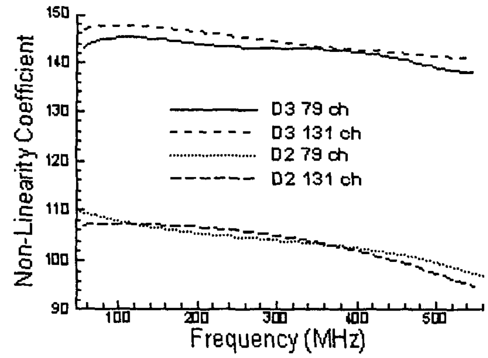

Fig. 5. Non-linearity coefficients (dB) with validation.

data, confirming not only the accuracy of the measurements, but the validity of the mathematical model used to derive the non-linearity constants. The discrepancy noted in Fig. 5 is within the measurement accuracy of about 2.5 dB over most of the band. All the data described so far is for the amplifier output leveled to 44 dBmV. With the corresponding constants for the 79-channel load (see Fig. 5), we have used the model in Sec. III and computed the CSO and CTB for a *12.5 dBmV up-tilted output*. The result, omitted for brevity, is then compared to the tilted-output *measured* data for the same channel loading. Again, we have observed corroboration within 2 dB, validating the hypothesis that D_2 and D_3 adequately account for the multiple beat products.

V. CONCLUSION

We have proposed a method for the analysis of intermodulation distortion in a multicarrier system, which exploits the relationship between measured non-linear performance and the signal characteristics of the second- and third-order distortion products. We have applied the method to determine the non-linearity constants of a CATV amplifier for two different channel loads, and demonstrated good corroboration.

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